Integrazione delle funzioni razionali fratte

Esercizi

n. 175 p. 192 (C_2)

$$\int \frac{2x^3-4x^2+3}{2x-4}\,dx = \left(\text{dividendo i polinomi}\right)$$

$$= \int \left(x^2 + \frac{3}{2x-4}\right) dx = \frac{x^3}{3} + \frac{3}{2}\ln|2x-4| + c$$

$$= \frac{x^3}{3} + \frac{3}{2}\ln|x-4| + c$$

n. 179

$$\int \frac{2x^3+18x+1}{x^2+9}\,dx = \int \left(2x + \frac{1}{x^2+9}\right) dx =$$

$$= x^2 + \frac{1}{3}\arctan\frac{x}{3} + c$$

n. 185

$$\int \frac{1}{(x+3)^2}\,dx = -\frac{1}{x+3} + c \qquad (\Delta = 0)$$

n. 187

$$\int \frac{1}{x^2-4x+4}\,dx = \int \frac{1}{(x-2)^2}\,dx = -\frac{1}{x-2} + c$$

n. 191

$$\int \frac{x-3}{x^2-10x+25}\,dx = \frac{1}{2}\int \frac{2x-6}{x^2-10x+25}\,dx =$$

$$= \frac{1}{2}\int \frac{2x-10+4}{x^2-10x+25}\,dx = \frac{1}{2}\int \frac{2x-10}{x^2-10x+25}\,dx +$$

$$+ \frac{1}{2}\int \frac{4}{(x-5)^2}\,dx = \frac{1}{2}\ln(x-5)^2 - \frac{2}{x-5} + c =$$

$$= \ln|x-5| - \frac{2}{x-5} + c$$

$$\left(\begin{array}{l} \text{Verifica:} \\ D\left(\ln|x-5| - \frac{2}{x-5} + c\right) = \frac{1}{x-5} + \frac{2}{(x-5)^2} = \frac{x-3}{(x-5)^2} \end{array} \right)$$

n. 194

$$\int \frac{1}{x^2+2x+6}\,dx = \quad (\Delta < 0)$$

$$= \int \frac{1}{(x+1)^2-1+6}\,dx = \int \frac{1}{(x+1)^2+5}\,dx =$$

$$= \frac{1}{\sqrt{5}}\int \frac{\frac{1}{\sqrt{5}}}{\left(\frac{x+1}{\sqrt{5}}\right)^2+1}\,dx = \frac{1}{\sqrt{5}} \arctan \frac{x+1}{\sqrt{5}} + c$$

n. 204

$$\int \frac{x}{2x^2+x+1}\,dx = \frac{1}{4}\int \frac{4x+1-1}{2x^2+x+1}\,dx =$$

$$= \frac{1}{4}\int \frac{4x+1}{2x^2+x+1}\,dx - \frac{1}{4}\int \frac{1}{2x^2+x+1}\,dx =$$

$$= \frac{1}{4}\ln\left(2x^2+x+1\right) - \frac{1}{8}\int \frac{1}{x^2+\frac{x}{2}+\frac{1}{2}}\,dx =$$

Calcolo l'ultimo integrale

$$\int \frac{1}{x^2+\frac{x}{2}+\frac{1}{2}}\,dx =$$

$$= \int \frac{1}{\left(x+\frac{1}{4}\right)^2-\frac{1}{16}+\frac{1}{2}}\,dx = \int \frac{1}{\left(x+\frac{1}{4}\right)^2+\frac{7}{16}}\,dx =$$

$$= \frac{16}{7}\int \frac{1}{\left(\frac{x+\frac{1}{4}}{\frac{\sqrt{7}}{4}}\right)^2+1}\,dx = \frac{16}{7}\int \frac{1}{\left(\frac{4x+1}{\sqrt{7}}\right)^2+1}\,dx =$$

$$= \frac{4}{\sqrt{7}}\arctan\left(\frac{4x+1}{\sqrt{7}}\right)+c$$

Tornando al calcolo initiale:

$$= \frac{1}{4}\ln\left(2x^2+x+1\right) - \frac{1}{2\sqrt{7}}\arctan\frac{4x+1}{\sqrt{7}} + c$$

n. 226

$$\int \frac{1}{x^2-2x}\,dx = \int \frac{1}{x(x-2)}\,dx = \quad (\Delta>0)$$

Scomposizione della frazione

$$\frac{1}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2} = \frac{A(x-2)+Bx}{x(x-2)} =$$

$$= \frac{Ax-2A+Bx}{x(x-2)} = \frac{x(A+B)-2A}{x(x-2)}$$

$$\begin{cases} A+B=0 \\ -2A=1 \end{cases} \quad \begin{cases} B=-A \\ A=-\frac{1}{2} \end{cases} \quad \begin{cases} B=\frac{1}{2} \\ A=-\frac{1}{2} \end{cases}$$

$$= \int -\frac{1}{2x}\,dx + \int \frac{1}{2(x-2)}\,dx =$$

$$= -\frac{1}{2}\ln|x| + \frac{1}{2}\ln|x-2| + c = \frac{1}{2}\ln\left|\frac{x-2}{x}\right| + c$$

n. 229

$$\int \frac{1}{x^2+3x+2}\,dx = \int \frac{1}{(x+2)(x+1)}\,dx = \int\left(-\frac{1}{x+2}+\frac{1}{x+1}\right)dx =$$

$$= -\ln|x+2| + \ln|x+1| + c = \ln\left|\frac{x+1}{x+2}\right| + c$$

n. 245

$$\int \frac{x+1}{x^3-6x^2+12x-8}\,dx = \int \frac{x+1}{(x-2)^3}\,dx =$$

Scomposizione della frazione

$$\frac{x+1}{(x-2)^3} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3} =$$

$$= A(x^2+4-4x) + Bx - 2B + C$$

$$\begin{cases} A=0 \\ B-4A=1 \\ 4A-2B+C=1 \end{cases} \qquad \begin{cases} A=0 \\ B=1 \\ C=3 \end{cases}$$

$$= \int \frac{1}{(x-2)^2}\,dx + \int \frac{3}{(x-2)^3}\,dx =$$

$$= -\frac{1}{x-2} - \frac{3}{2}\frac{1}{(x-2)^2} + C =$$

$$= \frac{1-2x}{2(x-2)^2} + c$$